The Surface Rejuvenation Theory of Wall Turbulence for Momentum, Heat and Mass Transfer: Application to Moderate and High Schmidt (Prandtl) Fluids

The surface rejuvenation model of the turbulent burst process, already developed for the momentum and heat transfer processes, is applied to the field of mass transfer. The range of applicability of the theory, previously shown to be valid for the Prandtl number range 0.7 to 64 is extended to Schmidt numbers of 6,500 without introducing any new parameters. Concentration and temperature profiles for high Schmidt and Prandtl numbers are in agreement with experimental data. Theoretical expressions for Sherwood and Stanton numbers are calculated and shown to be in agreement with experimental data over five orders of magnitude.

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SCOPE

The unsteady nature of wall turbulence was detected as early as 1932 by Fage and Townend. More recent studies indicate that the wall region in turbulent flows consists of numerous large-scale coherent structures which are intermittently rejected into the turbulent core. A model of wall turbulence was introduced in 1962 by Harriott which accounts for the major characteristics of the actual turbulent burst phenomenon. However this surface rejuvenation model involved lengthy Monte Carlo type numerical calculations, failed to provide a basis for establishing inputs for the key wall turbulence parameters, and lacked an interface with the turbulent core. More recently this modeling concept has been cast into a mathematically closed form and solved analytically for fluid flow and heat transfer by Thomas et al. (1975, 1980). In addition, the wall turbulence modeling parameters have been specified and the model has been interfaced with a classical model for the turbulent core for fluids with moderate values of Prandtl numbers. The objective of this paper is to extend this surface rejuvenation model of wall turbulence to mass and heat transfer for turbulent flow of fluids with large values of Schmidt number Sc or Prandtl number Pr. To test the model, predictions for Stanton number St and distribution in mean concentration and temperature are compared with experimental data for Sc and Pr in the range 0.5 to 10^4. In addition, a useful correlation is developed for St which applies across this wide range of conditions.

CONCLUSIONS AND SIGNIFICANCE

A turbulent burst or surface rejuvenation model has been developed in this study for mass or heat transfer to fluids with values of Sc or Pr up to 10^4. In this approach, the actual unsteady process that occurs between inrush and ejection phases of a burst event is modeled, with the approach distance distribution specified by Danckwerts (1951) experimental distribution. This model of wall turbulence has been coupled with the classical eddy diffusivity representation of the turbulent core to obtain predictions of ω^+, $\bar{\omega}$, and St for fully developed fully turbulent flow. Model closure involves the specification of the mean burst frequency $\bar{s}$ and the mean approach distance $\bar{H}$ on the basis of hydrodynamics, with no inputs being required for eddy diffusivity/mixing length or turbulent Schmidt number Sc_t within the critical wall region. The model predictions for ω^+ and $\bar{\bar{\omega}}$ are in basic agreement with the limited experimental data for high Schmidt numbers. This turbulent burst analysis gives rise to an expression for Stanton number of the form

$$St = \frac{0.023Re^{-0.2}}{1 + 1.93Re^{-0.1}(Sc^{2/3} - 1)}$$

which is in agreement with experimental data for Sc from 0.5 to 10,000. The analysis indicates that the thickness of the unreplenished layer of fluid existing at the surface is significant for values of Sc much greater than 5. For values of Sc less than about 5, this layer occupies less than 10% of the molecular wall region, such that the simple surface renewal model can be used.

Based on the results of this analysis and on earlier work, it is concluded that the surface rejuvenation model provides a useful

alternative approach to characterizing the wall region for moderate to high values of *Sc* and *Pr*. Because of the strong physical basis for the surface rejuvenation model, and as this approach involves a minimum level of empiricism, the theory is believed to provide a fundamental basis for generalization to other transport processes involving wall turbulence.

INTRODUCTION

The lack of a reliable theoretical model of wall turbulence is one of the main limitations of existing approaches to analyzing turbulent momentum, heat and mass transfer processes. This lack is clearly demonstrated by the multitude of models that presently exist to characterize this region, none of which has been generally adopted as a totally satisfactory explanation of the various phenomena involved. The study of alternative approaches recommended by Spalding (1978) appears warranted.

Various approaches, many of which have practical and conceptual limitations, have been used to model mass transfer near the wall include the following: 1. those based on the film theory or combinations with the film theory such as Toor and Marchello (1958); 2. those based on the arrival and departure of patches of fluid at the wall or adjacent thereto in which surface renewal is assumed to take place (Einstein and Li, 1956; Hanratty, 1956; Harriott, 1962; and 3., similar to (2) with respect to the patch concept, except that boundary-layer flow is assumed to develop within the patches (Ruckenstein, 1967; Pinczewski and Sideman, 1974). To supplement these modeling approaches, analysts sometimes resort to empirical curve fitting, as in the recent paper by Kader (1981), or employ analogies between mass, heat and momentum transfer, with the turbulent Schmidt number simply set equal to unity. These various approaches are summarized by Sideman and Pinczewski (1975) and Brodkey et al. (1978).

In the pioneering study of the behavior of flow in the immediate vicinity of the wall, Fage and Townend (1932) observed that turbulent fluctuations even occurred in the fluid in the viscous laminar sublayer region. In more recent studies, fluctuations have been observed in this region by many authors (e.g., Shaw and Hanratty, 1964; Kline et al., 1967; Grass, 1971). The fluctuations within the wall region appear to be associated with a turbulent burst phenomenon which is characterized by a mean burst frequency $\bar{s}$ and a mean distance $\bar{H}$ from the wall at which fluid is ejected.

The first model of wall turbulence to account for these factors was developed by Harriott (1962). However, this surface rejuvenation model involved lengthy Monte Carlo type numerical calculations and failed to provide a basis for establishing input for $\bar{s}$ and $\bar{H}$ and for interfacing with the turbulent core. A more efficient numerical scheme for accounting for the random variation in s and H was introduced in 1972 by Bullin and Dukler. Finally, the surface rejuvenation model was cast into the form of a partial differential equation and solved analytically in 1975 by Thomas et al. This work was followed up in a paper by Thomas (1980) on turbulent convection heat transfer for moderate Prandtl numbers $0.72 < Pr < 64$ in which $\bar{s}$ and $\bar{H}$ were established on the basis of hydrodynamic information and in which the surface rejuvenation model was interfaced with a classical model for the turbulent core. In the present paper this approach is extended to mass transfer for moderate to high values of Schmidt number.

THEORY

The turbulent burst process involves the intermittent exchange of fluid between the wall region and the turbulent core as pictured in Figure 1. The burst frequency s and the approach distance H are statistical in nature varying randomly about mean values $\bar{s}$ and $\bar{H}$. The analysis of this complex situation is made mathematically tractable by following the procedure formulated by Thomas et al. (1975).

Focusing attention on uniform property binary mass transfer in a fully developed, fully turbulent flow field and assuming negligible convective effects within the wall region, the unsteady mass transfer for the period between inrush and ejection is approximated by

$$\frac{\partial \omega}{\partial \theta} = D \frac{\partial^2 \omega}{\partial y^2} \tag{1}$$

where ω is the mass fraction of the diffusing species, D is the diffusivity, and θ is the age of the event. The initial and boundary conditions are given by

$$\omega = \omega_{tc}[U(y-H)] + r(y)[1 - U(y-H)] \text{ at } \theta = 0 \tag{2}$$

$$\omega = \omega_j \text{ at } y = y_j \tag{3}$$

and

$$\omega = \omega_o \text{ at } y = 0 \tag{4}$$

for specified wall concentration, or

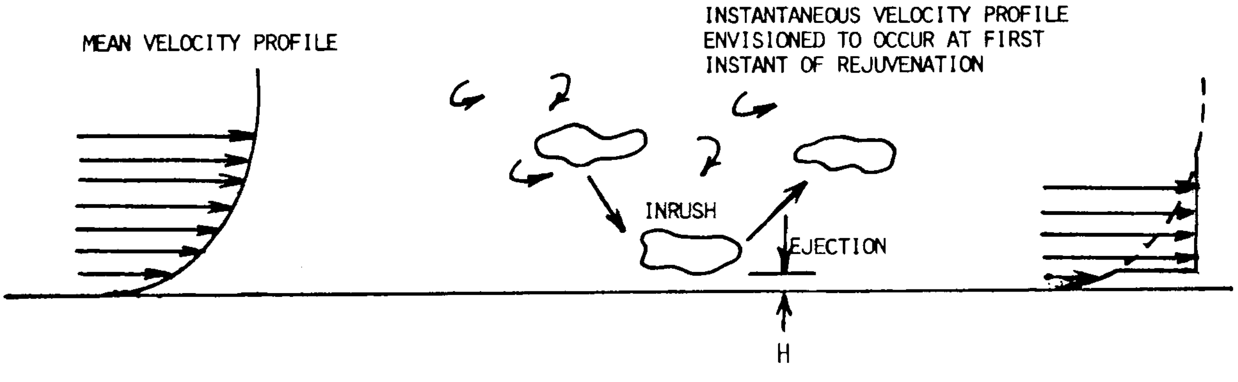

Figure 1. Pictorial diagram of the turbulent burst process.

$$n_A = -\rho D \left.\frac{\partial \omega}{\partial y}\right|_{y=0} \quad (5)$$

for specified wall mass flux n_A; ω_{ic} is the concentration of the inrushing fluid, $r(y)$ is the instantaneous concentration distribution within the wall region at the first instant of inrush or renewal, $U(y - H)$ is a unit step function, H is the approach distance to the surface, ω_j is the condition at the interface between the wall region and the turbulent core, n_A is the mass flux, and ρ is the mass density. The statistical properties θ, s, H, $r(y)$, and ω_{ic} are given by $\phi(\theta,\bar{s})$, $P_\theta(\theta)$, $P_H(H)$, $P_r(r)$, and $P_{\omega_{ic}}(\omega_{ic})$, respectively. The age, frequency, and approach distance distributions are approximated by the convenient Danckwerts (1951) exponential distribution.

$$\phi(\theta,\bar{s}) = \bar{s}\exp(-\bar{s}\theta) \quad (6)$$

$$P_\theta(\theta) = -\frac{1}{\bar{s}}\frac{d\phi(\theta,\bar{s})}{d\theta} = \bar{s}\exp(-\bar{s}\theta) \quad (7)$$

$$P_H(H) = \frac{1}{\overline{\overline{H}}}\exp\left(-\frac{H}{\overline{\overline{H}}}\right) \quad (8)$$

The instantaneous transport equations are now transformed into the mean domain. By definition, the mean concentration is expressed by

$$\overline{\omega} = \int_o^\infty P_H(H)\int_o^\infty P_{\omega_{ic}}(\omega_{ic}) \times \int_o^\infty P_r(r)\int_o^\infty \omega\phi(\theta,\bar{s})d\theta\, dr d\omega_{ic} dH \quad (9)$$

Accordingly, Eqs. 1 to 4 are transformed into the mean domain by multiplying each term by $\phi(\theta,\bar{s})d\theta$, $P_r(r)dr$, $P_{\omega_{ic}}(\omega_{ic})d\omega_{ic}$, and $P_H(H)dH$ and then integrating.

Operating first with respect to θ, Eqs. 1 to 4 give

$$\tilde{\omega} - r(y)[1 - U(y-H)] - \omega_{ic}[U(y-H)] = \frac{D}{\bar{s}}\frac{d^2\tilde{\omega}}{dy^2} \quad (10)$$

$$\tilde{\omega} = \tilde{\omega}_j \text{ at } y = y_j \quad (11)$$

and

$$\tilde{\omega} = \tilde{\omega}_o \text{ at } y = 0 \quad (12)$$

where

$$\tilde{\omega} = \bar{s}\int_o^\infty \omega\exp(-\bar{s}\theta)d\theta = \bar{s}\mathcal{L}(\omega) \quad (13)$$

and $\mathcal{L}(\omega)$ represents the Laplace transform. Operating next with respect to $P_r(r)$ and $P_{\omega_{ic}}(\omega_{ic})$, these equations take the form

$$\hat{\omega} - \bar{r}(y)[1 - U(y-H)] - \overline{\omega}_{ic}[U(y-H)] = \frac{D}{\bar{s}}\frac{d^2\hat{\omega}}{dy^2} \quad (14)$$

$$\hat{\omega} = \hat{\omega}_j \text{ at } y = y_j \quad (15)$$

and

$$\hat{\omega} = \hat{\omega}_o \text{ at } y = 0 \quad (16)$$

where

$$\hat{\omega} = \int_o^\infty P_{\omega_{ic}}(\omega_{ic})\int_o^\infty P_r(r)\tilde{\omega} dr d\omega_{ic} \quad (17)$$

Finally, operating with respect to H, Eqs. 14 to 17 are transformed into the mean domain

$$\overline{\omega} - \bar{r}(y)\exp\left(-\frac{y}{\overline{H}}\right) - \overline{\omega}_{ic}\left[1 - \exp\left(-\frac{y}{\overline{H}}\right)\right] = \frac{D}{\bar{s}}\frac{d^2\overline{\omega}}{dy^2} \quad (18)$$

$$\overline{\omega} = \overline{\omega}_j \text{ at } y = y_j \quad (19)$$

and

$$\overline{\omega} = \overline{\omega}_o \text{ at } y = 0 \quad (20)$$

where

$$\overline{\omega} = \int_o^\infty \frac{1}{H}\exp\left(-\frac{H}{\overline{\overline{H}}}\right)\hat{\omega} dH \quad (21)$$

Whereas the mean concentration profile is given by Eq. 9, the mean initial concentration profile $\bar{r}(y)$ can be expressed by

$$\bar{r}(y) = \int_o^\infty P_H(H)\int_o^\infty P_{\omega_{ic}}(\omega_{ic}) \times \int_o^\infty P_r(r)\int_o^\infty \omega P_\theta(\theta)d\theta dr d\omega_{ic} dH \quad (22)$$

where the frequency distribution $P_\theta(\theta)$ is given by Eq. 7. Because the distribution functions $\phi(\theta,\bar{s})$ and $P_\theta(\theta)$ are identical, Eqs. 9 and 22 are equivalent such that

$$\overline{\omega} = \bar{r}(y) \quad (23)$$

using Eq. 23, Eq. 18 may be written as

$$(\overline{\omega} - \overline{\omega}_{ic})\left[1 - \exp\left(-\frac{y}{\overline{H}}\right)\right] = \frac{D}{\bar{s}}\frac{d^2\overline{\omega}}{dy^2} \quad (24)$$

The modeling equations for mean mass transfer within the wall region, Eqs. 19, 20, and 24 involve the turbulent transport parameters $\bar{s}$, $\overline{H}$, $\overline{\omega}_{ic}$ and $\overline{\omega}_j$. These equations are of the same form as those developed for momentum and energy transfer (Thomas et al., 1975; Thomas, 1980). Because of the very small thickness of the diffusion wall region for moderate to high Sc, the parameter $\overline{\omega}_j$ is eliminated by relaxing the constraint imposed by Eq. 19 and allowing $\overline{\omega}$ to approach $\overline{\omega}_{ic}$ as y becomes large; that is

$$\overline{\omega} = \overline{\omega}_{ic} \text{ as } y \to \infty \quad (25)$$

The analytical solution to ordinary differential equations of the form of Eq. 24 have been obtained by utilizing the substitutions $\psi = \overline{\omega} - \overline{\omega}_{ic}$, $Z = \exp[-y/(2\overline{H})]$ and $\xi = 2Z\overline{H}\bar{s}/D$ (Thomas et al., 1975). The final solution for the mean concentration distribution for the case in which Eq. 25 is employed takes the form

$$\frac{\overline{\omega} - \overline{\omega}_o}{\overline{\omega}_{ic} - \overline{\omega}_o} = 1 - \frac{J_{2\lambda}[2\lambda\exp(-y/(2\overline{H}))]}{J_{2\lambda}(2\lambda)} \quad (26)$$

where $\lambda = \overline{H}\sqrt{\bar{s}/D} = \gamma\sqrt{Sc}$, $\gamma = \overline{H}\sqrt{\bar{s}/\nu}$, and $J_{2\lambda}$ is a Bessel function of the first kind and 2λ order. The mean wall mass flux $\bar{n}_A$ is given by

$$\bar{n}_A = -\rho D\left.\frac{d\overline{\omega}}{dy}\right|_{y=0} \quad (27)$$

$$= \rho(\overline{\omega}_o - \overline{\omega}_{ic})\sqrt{\bar{s}D}\frac{J_{2\lambda-1}(2\lambda) - J_{2\lambda+1}(2\lambda)}{2J_{2\lambda}(2\lambda)} \quad (28)$$

By combining Eqs. 26 and 27, an expression is obtained for the dimensionless concentration profile ω^+ of the form

$$\omega^+ = \frac{(\overline{\omega}_o - \overline{\omega})\rho U^*}{\bar{n}_A} \quad (29)$$

$$= 2H^+\frac{Sc}{\lambda}\frac{J_{2\lambda}(2\lambda) - J_{2\lambda}[2\lambda\exp(-y^+/(2H^+))]}{J_{2\lambda-1}(2\lambda) - J_{2\lambda+1}(2\lambda)} \quad (30)$$

where U^* is the friction velocity ($\equiv \sqrt{\overline{\tau}_o/\rho}$), $y^+ = yU^*/\nu$ and $H^+ = \overline{H}U^*/\nu$.

The analogous equations for momentum and heat transfer are (Thomas et al., 1975; Thomas, 1980)

$$\frac{\bar{u}}{\overline{U}_{ic}} = 1 - \frac{J_{2\lambda}[2\lambda\exp(-y/2\overline{H})]}{J_{2\gamma}(2\gamma)} \quad (31)$$

and

$$\frac{\overline{T} - \overline{T}_o}{\overline{T}_{ic} - \overline{T}_o} = 1 - \frac{J_{2\beta}[2\beta\exp(-y/2\overline{H})]}{J_{2\beta}(2\beta)} \quad (32)$$

where $\beta = \overline{H}\sqrt{\bar{s}/\alpha} = \gamma\sqrt{Pr}$, and $\overline{U}_{ic}$ and $\overline{T}_{ic}$ are analogous to

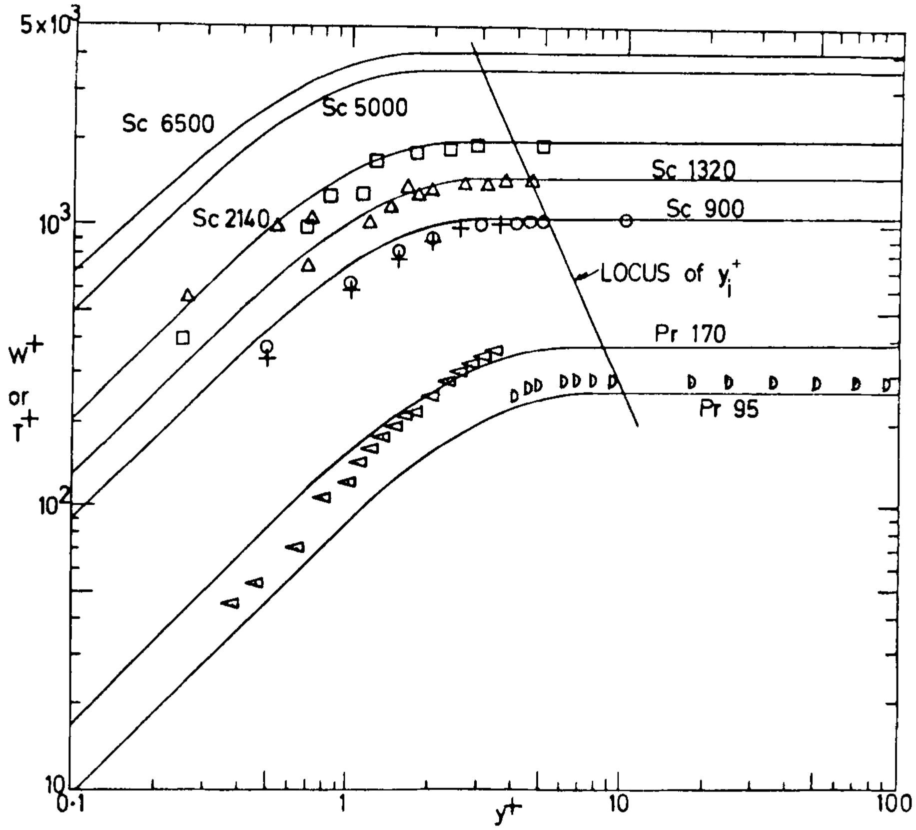

Figure 2. Variation of ω^+ or T^+ with y^+ for high Sc or Pr numbers.

	Condition	Data from
O	Sc = 900, Re = 9,700	Lin et al. (1953)
+	Sc = 900, Re = 12,400	Lin et al. (1953)
Δ	Sc = 1,320, Re = 25,000	Hiby & Flender (1981)
□	Sc = 2,140, Re = 10,000	Hiby & Flender (1981)
◁	Pr = 170	Kader (1981)
D	Pr = 95	Kader (1981)

———— Theoretical curves from Eqs. 30 and 42

Locus y_i^+ = Intersection of wall and turbulent core equations

$\overline{\omega}_{tc}$. The mean wall shear stress $\overline{\tau}_o$ and the mean wall heat flux q_o are given by

$$\overline{\tau}_o = \mu \overline{U}_{tc} \sqrt{\frac{\overline{s}}{\nu}} \frac{J_{2\gamma-1}(2\gamma) - J_{2\gamma+1}(2\gamma)}{2J_{2\gamma}(2\gamma)} \tag{33}$$

$$\overline{q''_o} = k(\overline{T}_o - \overline{T}_{tc}) \sqrt{\frac{\overline{s}}{\alpha}} \frac{J_{2\beta-1}(2\beta) - J_{2\beta+1}(2\beta)}{2J_{2\beta}(2\beta)} \tag{34}$$

By combining Eqs. 31 and 33, and Eqs. 32 and 34, respectively, expressions are obtained for the dimensionless velocity and temperature profiles u^+ and T^+.

$$u^+ = \frac{\overline{u}}{U^*} = \frac{2H^+}{\gamma} \times \left\{ \frac{J_{2\gamma}(2\gamma) - J_{2\gamma}[2\gamma \exp(-y^+/(2H^+))]}{J_{2\gamma-1}(2\gamma) - J_{2\gamma+1}(2\gamma)} \right\} \tag{35}$$

$$T^+ = \frac{(\overline{T}_o - \overline{T})}{\overline{q''_o}} \rho c_P U^* \tag{36}$$

$$= 2H^+ \frac{Pr}{\beta} \frac{J_{2\beta}(2\beta) - J_{2\beta}[2\beta \exp(-y^+/(2H+))]}{J_{2\beta-1}(2\beta) - J_{2\beta+1}(2\beta)} \tag{37}$$

It should be noted that the above wall laws for ω^+, u^+ and T^+ only involve the two hydrodynamic modeling parameters s^+ and H^+.

Turbulent Core

To analyze the mass transfer within the turbulent core the classical mixing length/eddy diffusivity approach is adopted. The mean mass flux is expressed in terms of the mass eddy diffusivity ϵ_D by

$$\overline{n}_A = -(\epsilon_D + D)\frac{d\overline{\omega}}{dy} \tag{38}$$

Introducing the dimensionless wall variables ω^+ and y^+, this equation becomes

$$\left(\frac{\epsilon_D}{\nu} + \frac{1}{Sc}\right)\frac{d\omega^+}{dy^+} = \frac{\overline{n}_A}{\overline{n}_{Ao}} \tag{39}$$

where $\overline{n}_{Ao}$ is the mean flux at the wall. The turbulent Schmidt number Sc_t is generally used to express ϵ_D in terms of the momentum eddy diffusivity ϵ_M; that is,

$$Sc_t = \frac{\epsilon_M}{\epsilon_D} \tag{40}$$

In the intermediate region, ϵ_M can be approximated by

$$\frac{\epsilon_M}{\nu} = \kappa y^+ \tag{41}$$

where $\kappa \simeq 0.4$. Combining Eqs. 39, 40, and 41, and assuming that

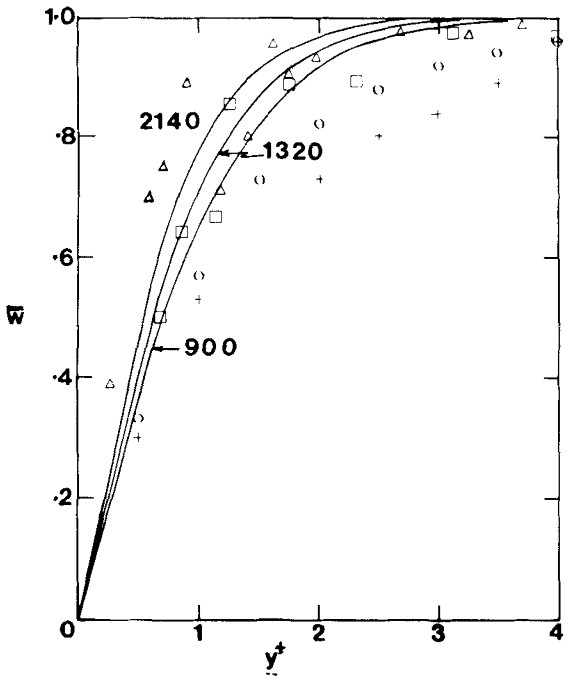

Figure 3. Variation of $\overline{\omega}$ with y^+ for high Sc numbers.

	Condition	Data from
○	$Sc = 900, Re = 9{,}700$	Lin et al. (1953)
+	$Sc = 900, Re = 12{,}400$	Lin et al. (1953)
△	$Sc = 1{,}320, Re = 25{,}000$	Hiby & Flender (1981)
□	$Sc = 2{,}140, Re = 10{,}000$	Hiby & Flender (1981)
———	Theoretical curves from Eq. 30	

$\overline{n}_A/\overline{n}_{Ao} \simeq 1$ and Sc_t is approximately uniform throughout the turbulent core, the solution for ω^+ in the intermediate region for moderate to high values of Sc is given by

$$\omega^+ = \frac{Sc_{t\infty}}{\kappa} \ln y^+ + A \tag{42}$$

where A is a function of Sc. The turbulent Schmidt number $Sc_{t\infty}$ is usually set equal to a value of unity.

An expression can be developed for ω^+ within the outer region by using either of several standard approximations that are available for ϵ_M and by accounting for the variation in $\overline{n}_A/\overline{n}_{Ao}$. Alternatively, ω^+ can simply be approximated by Eq. 42. To simplify the presentation the second approach is used.

Similar laws have been developed for u^+ and T^+ in the turbulent core; that is

$$u^+ = \frac{1}{\kappa} \ln y^+ + C \tag{43}$$

$$T^+ = \frac{Pr_{t\infty}}{\kappa} \ln y^+ + B \tag{44}$$

where C is constant, $Pr_{t\infty}$ is the turbulent Prandtl number, and B is a function of Pr.

To complete the analysis, Eqs. 30 and 42 are matched at the point where continuity is maintained in $d\omega^+/dy^+$. A prerequisite to this step is the specification of the hydrodynamic modeling parameters s^+ and H^+.

The dimensionless mean approach distance H^+ is set equal to 5.0 on the basis of experimental flow visualization measurements by Popovich and Hummel (1967). The dimensionless frequency s^+ is taken from the momentum analysis by Thomas (1980); that is $\gamma = \overline{H}\sqrt{\overline{s}/\nu} = 0.433$ or $s^+ = \overline{s}\nu/U^{*2} = 0.0075$. The coupling of the solutions for the wall region and the turbulent core are performed numerically. This step gives predictions for the parameter A as a function of Sc.

RESULTS

The thermal and momentum analysis of Thomas et al. (1975, 1980) established the validity of the burst frequency model for these transport processes. However since the maximum Prandtl number in the thermal analysis was ony 64, a test of the model validity at higher Prandtl or Schmidt numbers is of interest. In the present study, Eq. 30 is solved for Schmidt numbers as high as 6,500. Because the Bessel functions of the first kind involve 80 to 100 terms for such high values of Sc, Eq. 30 is calculated by using the traditional nested numerical procedure employing double precision arithmetic to avoid round-off error. An alternative procedure using asymptotic expansions has also been successfully employed up to Schmidt numbers of 15,000.

Calculations for ω^+ obtained on the basis of Eqs. 30 and 42 are compared in Figure 2 with experimental data for Sc (and Pr) ranging from 95 to 2,140. [The data for concentration profile $\overline{\omega}$ for high Sc by Flender and Hiby (1981) and Lin et al. (1953) were transformed by the use of the familiar Friend and Metzner (1958) correlation for St.] The agreement between the theory and experiment is quite good over this broad range of conditions. The theoretical calculations and experimental data are shown in terms of $\overline{\omega}$ in Figure 3.

Calculations for the interface parameter y_i^+ are shown in Figures 2 and 4. This parameter decreases from a value of about 45 at $Sc = 1$ to 10 at $Sc = 100$, and to 3 at $Sc = 6{,}000$. Thus, the unreplenished layer of fluid represents a significant fraction of the concentration wall region for fluids with large values of Sc. Furthermore, the significance of the statistical nature of the approach distance H is evidenced by the fact that y_i^+ is actually less than $H^+ (\overline{H}U^*/\nu = 5)$ for fluids with very high values of Sc.

Calculations for the parameter A are shown in Figure 4. These results are quite well represented by

$$A = 12.7(Sc^{2/3} - 1) + 5 \tag{45}$$

An equation of the same form has been recommended for the thermal parameter B by Petukhov (1970). Figure 4 also shows calculations reported by Thomas (1980) for B for Pr up to 64.

Using Eq. 42 to approximate ω^+ across the flow field and setting $Sc_t = 1$, an expression is obtained for the Stanton number St of the form

$$St = \frac{f/2}{1 + \sqrt{f/2}(A - 5)} \tag{46}$$

With A given by Eq. 45, this equation becomes

$$St = \frac{f/2}{1 + 12.7\sqrt{f/2}(Sc^{2/3} - 1)} \tag{47}$$

To express St in terms of Reynolds number, the friction factor is approximated by the Blasius equation for the friction factor, with the result

$$St = \frac{0.023Re^{-0.2}}{1 + 1.93Re^{-0.1}(Sc^{2/3} - 1)} \tag{48}$$

For large Sc, this can be simplified to

$$St = 0.0119Re^{-0.1}Sc^{-2/3} \tag{49}$$

with an accuracy better than 5% for $Sc > 150$. Equation 47 is of the same form as the expression for Nusselt number developed by

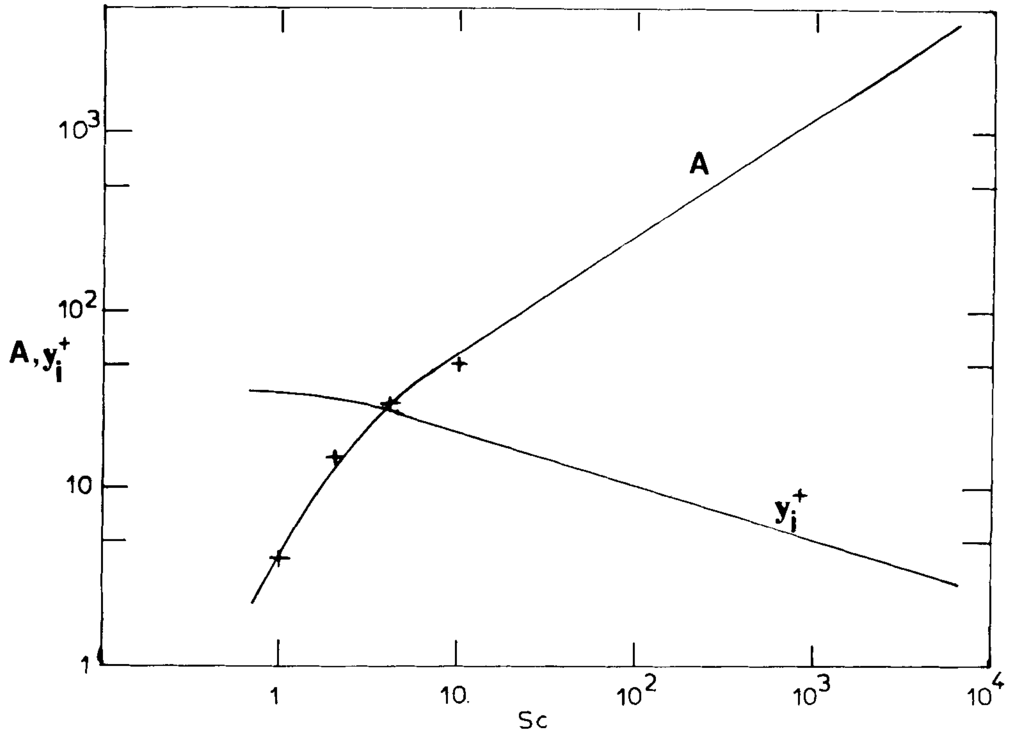

Figure 4. Plot of parameter A and y_i^+ vs. Sc. Equation 45 analogous to thermal parameter B of Petukhov (1970) is indistinguishable from values of A given. + = values of thermal parameter B of Thomas (1980).

Petukhov (1970) on the basis of classical eddy diffusivity and turbulent Prandtl number inputs.

Equation 47 is compared with experimental data and the empirical correlation of Friend and Metzner (1958) in Figure 5. The agreement is exceptional for values of Sc ranging from about 0.5 to 10,000.

ACKNOWLEDGMENT

The authors wish to acknowledge the support of the University of Petroleum and Minerals while conducting this research.

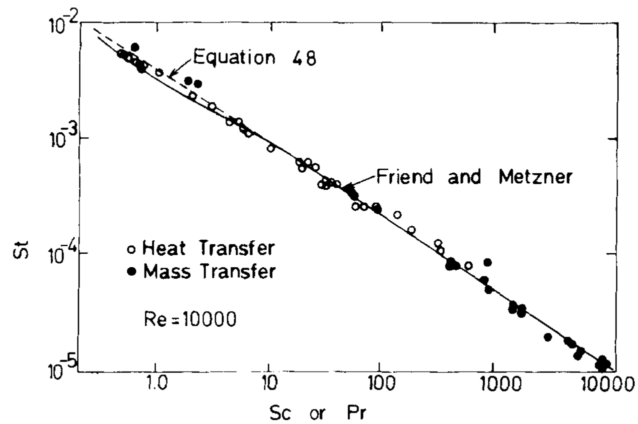

Figure 5. Variation of Stanton number St with Sc or Pr for flow of gases and liquids in tubes at Re 10,000. --- Theoretical curve for Eq. 48. —— Empirical curve of Friend and Metzner (1958).

NOTATION

A = parameter defined by Eq. 42
B = parameter defined by Eq. 44
C = constant defined by Eq. 43
D = diffusivity
f = friction factor
H = instantaneous approach distance
$J_{2\lambda}$ = Bessel function of 1st kind, order 2λ
k = thermal conductivity
k^+ = k_c/U^*, dimensionless mass transfer coefficient
n = mass flux
Pr = Prandtl number
P_r = statistical distribution for $r(y)$
P_H = statistical distribution for H
$P_{\omega_{tc}}$ = statistical distribution for ω
$\overline{q}''_o$ = mean wall heat flux
$r(y)$ = instantaneous concentration distribution within wall region
Re = Reynolds number
s = burst frequency
Sc = Schmidt number
Sh = Sherwood number
St = Stanton number
u = instantaneous velocity distribution
U^* = friction velocity $(=\sqrt{\overline{\tau}_o/\rho})$
U = unit step function
y = coordinate normal to wall
Z = transform, $\exp[-y/(2\overline{H})]$

Greek Letters

α = thermal diffusivity
β = transform, $\equiv \overline{H}\sqrt{\overline{s}/\alpha}$

γ = transform, $\equiv \overline{H}\sqrt{\overline{s}/\nu}$
ϵ_D = turbulent eddy diffusivity
ϵ_M = momentum diffusivity
θ = age of event
κ = turbulence constant ($\simeq 0.40$)
λ = transform, $\equiv \overline{H}\sqrt{\overline{s}/D}$
μ = molecular viscosity
ν = kinematic viscosity
ξ = transform, $= 2Z\overline{H}\overline{s}/D$
ρ = density
$\overline{\tau_o}$ = mean wall stress
ϕ = statistical distribution for θ
ψ = transform $= \overline{\omega} - \overline{\omega}_{ic}$
ω = mass fraction

Subscripts and Superscripts

A = species A
A_o = species A at wall
D = mass diffusivity
ic = inrushing fluid
j = interface coordinate between wall and core
o = wall conditions
t = turbulent
$+$ = dimensionless wall coordinate
$=$ = normalized coordinate

LITERATURE CITED

Brodkey, R. S., K. N. McKelvey, and H. C. Hershey, "Mass Transfer at the Wall as a Result of Coherent Structures in a Turbulently Flowing Liquid," *Int. J. Heat Mass Transfer*, **21**, 593 (1978).

Bullin, J. A., and A. E. Dukler, "Random Eddy Models for Surface Renewal: Formulation as a Stochastic Process," *Chem. Eng. Sci.*, **27**, 439 (1972).

Danckwerts, P. V., "Significance of Liquid Film-Coefficients in Gas Absorption," *Ind. Eng. Chem.*, **43**, 1,460 (1951).

Einstein, H. A., and H. Li, "The Viscous Sublayer Along a Smooth Boundary," *Trans. Am. Soc. Civil Eng.*, **82**, 293 (1956).

Fage, A., and H. C. Townend, "An Examination of Turbulent Flow with a Microscope," *Proc. Roy. Soc. London*, **135A**, 656 (1932).

Flender, J. F., and J. W. Hiby, "Investigation of Solid/Liquid Mass Transfer by a Photometric Method," *Ger. Chem. Eng.*, **4**, 370 (1981).

Friend, W. L., and A. B. Metzner, "Turbulent Heat Transfer Inside Tubes and the Analogy, Among Heat, Mass and Momentum Transfer," *AIChE J.*, **4**, 393 (1958).

Grass, A. J., "Structural Features of Turbulent Flow Over Smooth and Rough Boundaries," *J. Fluid Mech.*, **50**, 233 (1971).

Hanratty, T. J., "Turbulent Exchange of Mass and Momentum with a Boundary," *AIChE J.*, **2**, 359 (1956).

Harriott, P., "A Random Eddy Modification of the Penetration Theory," *Chem. Eng. Sci.*, **17**, 149 (1962).

Higbie, R., "The Rate of Absorption of a Pure Gas into a Still Liquid During Short Periods of Exposure," *Trans. Am. Inst. Chem. Eng.*, **31**, 65 (1935).

Kader, B. A., "Temperature and Concentration Profiles in Fully Turbulent Boundary Layers," *Int. J. Heat Mass Transfer*, **24**, 1,541 (1981).

Kline, S. J., et al., "The Structure of Turbulent Boundary Layers," *J. Fluid Mech.*, **30**, 741 (1967).

Lin, C. S., R. W. Moulton, and G. L. Putnam, "Mass Transfer Between Solid Wall and Fluid Streams: Mechanism and Eddy Distribution Relationships in Turbulent Flow," *IEC*, **45**, 636 (1953).

Petukhov, B. S., "Heat Transfer and Fraction in Turbulent Pipe Flow with Variable Physical Properties," *Adv. Heat Trans.*, **6**, 503 (1970).

Pinczewski, W. V. and S. Sideman, "A Model for Mass (Heat) Transfer in Turbulent Tube Flow; Moderate and High Schmidt (Prandtl) Numbers," *Chem. Eng. Sci.*, **29**, 1,969 (1974).

Popovich, A. T. and R. L. Hummel, "Experimental Study of the Viscous Sublayer in Turbulent Pipe Flow," *AIChE J.*, **13**, 854 (1967).

Prandtl, L., "Bemerkung ubar den Warmeubergang in Rohr," *Physikalische Zeit.*, **29**, 487 (1928).

Reynolds, O., *Proc. Manchester Lit. Phil. Soc.*, **14**, 7 (1874); reprinted in "Papers on Mechanical and Physical Subjects," **1**, 81, Cambridge, NY (1900).

Ruckenstein, E., "On Solid-Liquid Mass Transfer in Turbulent Pipe Flow," **22**, 474 (1967).

Shaw, P. V., and T. J. Hanratty, "Fluctuations in the Local Rate of Turbulent Mass Transfer to a Pipe Wall," *AIChE J.*, **10**, 475 (1964).

Sherwood, T. K., R. L. Pigford, and C. R. Wilke, "Mass Transfer," McGraw-Hill, New York (1975).

Spalding, D. B., "Turbulence Models for Heat Transfer," *Proc. 6th Int. Heat Trans. Conf.*, **6**, 33 (1978).

Thomas, L. C., P. J. Gingo, and B. T. F. Chang, "The Surface Rejuvenation Model for Turbulent Convective Transport; An Exact Solution," *Chem. Eng. Sci.*, **30**, 1,239 (1975).

Thomas, L. C., "A Surface Rejuvenation Model of Wall Turbulence: Inner Laws for u^+ and T^+," *Int. J. Heat Mass Transfer*, **23**, 1,099 (1980).

Toor, H. L., and J. M. Marchello, "Film Penetration Model for Heat and Mass Transfer," *AIChE J.*, **4**, 97 (1958).

Von Karmon, Th., "The Analogy Between Fluid Friction and Heat Transfer," *Trans. ASME*, **61**, 705 (1939).

Manuscript received June 3, 1983; revision received Nov. 5 and accepted Nov. 24, 1984.